An Interactive Introduction to Search and Matching Models

A Large-Firm Approach for Undergraduate Teaching

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Outline

Two facts that should not coexist

- Newspapers report millions of unemployed workers . . .
- . . . and on the same page, firms unable to fill open positions.

In a frictionless market this would be a contradiction.
In the real world it is the rule.

Why?

Hiring is not instantaneous. Workers must search, apply, interview. Firms must post vacancies, screen, negotiate.
Matching takes time and resources $\Rightarrow$ unemployment and vacancies coexist.

See it in real data: U.S. BLS interactive Beveridge curve, updated monthly $\rightarrow$ bls.gov.

What this lecture covers

A simple version of the canonical search-and-matching model (Diamond–Mortensen–Pissarides, 2010 Nobel Prize).

By the end of the lecture you should be able to:

- explain why u and v coexist in equilibrium;
- locate equilibrium graphically as the intersection of two curves;
- perform comparative statics for the seven structural parameters;
- run policy experiments using a live Python simulator.

Setup: large-firm formulation, continuous time, constant productivity.

Methodological note: the model is dynamic at the *aggregate-flow level* (differential equations for stocks); the firm's hiring problem is solved as a steady-state *reduced-form* optimization, not as a fully intertemporal dynamic program with asset values. Same answer in steady state, accessible without Bellman equations.

Environment

Continuous time. Unit mass of identical workers. Search frictions in the labor market.

- Each worker is either employed or unemployed.
- Aggregate employment rate e, unemployment rate u, with $e + u = 1$.
- Labor-market tightness: $\theta = v/u$, where v is aggregate vacancies.

Notation. Lowercase = aggregates (e, u, v). The matching function uses these. Uppercase E, V for the firm appears in the appendix.

Why a representative firm?

Classical Pissarides: many small one-job firms with free entry. Here: **one representative multi-worker firm** that posts many vacancies at once. Same equilibrium under our assumptions, closer to real employer behavior.

The matching function

Cobb–Douglas matching technology:

$$M(u, v) \;=\; \phi\, u^{1-\alpha}\, v^{\alpha}, \qquad \phi > 0,\;\; \alpha \in (0, 1).$$

Define labor-market tightness: $\theta \equiv v/u$.

Two derived rates:

Job-finding rate (workers):

$$p(\theta) \;=\; \frac{M}{u} \;=\; \phi\, \theta^{\alpha}$$

$\uparrow$ in θ: tight markets help workers.

Vacancy-filling rate (firms):

$$q(\theta) \;=\; \frac{M}{v} \;=\; \phi\, \theta^{-(1-\alpha)}$$

$\downarrow$ in θ: tight markets hurt firms.

The central tension

Same variable θ helps one side and hurts the other. Identity: $p(\theta) = \theta\, q(\theta)$.

Firms (aggregate view)

Aggregate behavior of the firm sector:

- produces $Y = A\,e$ (constant productivity per worker);
- pays wage w to each employed worker;
- posts v vacancies in total at flow cost c each;
- loses workers at separation rate s.

Aggregate flow profits:

$$\Pi \;=\; A\,e \;-\; w\,e \;-\; c\,v.$$

Aggregate employment evolves: $\dot{e} = q(\theta)\,v - s\,e.$

Firm-level setup: use uppercase E, V (with $e = N_f \cdot E$, $v = N_f \cdot V$ across N_f identical firms). The firm problem is solved as a *steady-state reduced form*, not as a Bellman dynamic program — same answer in steady state. See Appendix A.

Hiring math

Posting one vacancy:

- costs c per unit of time,
- gets filled at rate $q(\theta)$.

$\Rightarrow$ **expected cost of hiring one worker** $= c/q(\theta)$.

Once filled, a job lasts until destruction at rate s:
$\Rightarrow$ **expected job duration** $= 1/s$.

In steady state the firm posts vacancies until:

flow surplus per worker = separation rate × expected hiring cost.

The job creation condition

JC condition

$$\boxed{A - w \;=\; \frac{s\,c}{q(\theta)}}$$

Solving for w gives the job creation curve in (θ, w) space:

$$w^{JC}(\theta) \;=\; A \;-\; \frac{s\,c}{q(\theta)}.$$

- Downward-sloping in θ: tighter market $\Rightarrow$ vacancies harder to fill $\Rightarrow$ wage must drop for firm to break even.
- Higher $A \Rightarrow$ JC shifts up.
- Higher s or $c \Rightarrow$ JC shifts down.

Why does s multiply $c/q(\theta)$? Because filled jobs separate at rate s, the firm must repeatedly re-hire over time. The relevant cost of holding one worker is the per-period flow cost of *re-hiring*, $s \cdot c/q(\theta)$.

(Full derivation: Appendix A of the paper.)

Nash bargaining

When a worker and a firm meet, the match generates a surplus that did not exist before.

Outside options:

- Worker: $b =$ unemployment benefit $+$ value of leisure.
- Firm: must reopen vacancy at expected cost $c\,\theta$.

The Nash bargain splits the surplus with worker weight $\beta \in (0, 1)$:

$$\max_{w} \, (w - b)^{\beta} \left[A - w + c\,\theta \right]^{1-\beta}.$$

Solution method: take logs, differentiate, set to zero, solve for w.

Reduced-form caveat: $A - w + c\theta$ represents the firm's surplus including recruiting-cost savings; it is not literally a Bellman continuation value. The full asset-value derivation (Pissarides, 2000) gives the same wage in steady state.

The wage curve

Wage curve (WC)

$$\boxed{w^{WC}(\theta) \;=\; (1-\beta)\,b \;+\; \beta\,(A + c\,\theta)}$$

A convex combination of:

- the worker's outside option b, weighted by $1-\beta$;
- the firm's full productive value $A + c\,\theta$, weighted by β.

Interpretation of $c\,\theta$: in a tight market, the firm has been *saving* on hiring costs $\Rightarrow$ worker can credibly demand a share.

Upward-sloping in θ.
Limits: $\beta = 0 \Rightarrow w = b$; $\quad \beta = 1 \Rightarrow w = A + c\,\theta$.

Two curves, one intersection

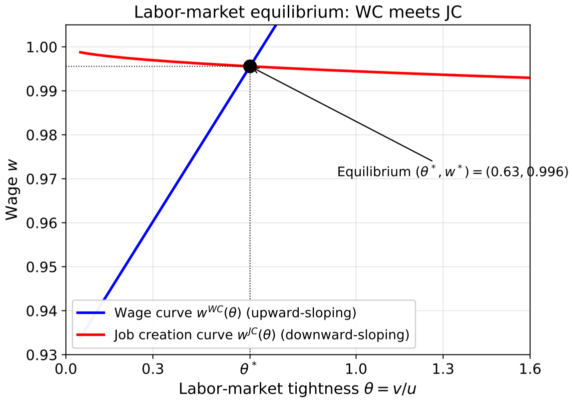
WC curve (blue): upward-sloping in θ.

JC curve (red): downward-sloping in θ.

$\Rightarrow$ **unique intersection** whenever $A > b$.

The intersection (θ^*, w^*) is the equilibrium.

Once θ^* is known, u^* and v^* follow.



Equilibrium tightness

Setting $w^{WC}(\theta) = w^{JC}(\theta)$:

$$(1-\beta)\,b \;+\; \beta\,(A + c\,\theta) \;=\; A \;-\; \frac{s\,c}{q(\theta)}.$$

- One equation in one unknown θ.
- Nonlinear, but easy to solve numerically (one line of `scipy.brentq`).
- Unique solution θ^* whenever $A > b$.

At the benchmark calibration:

$$\theta^* \approx 0.635, \quad w^* \approx 0.996.$$

Steady-state unemployment

Unemployment dynamics:

$$\dot{u} \;=\; \underbrace{s\,e}_{\text{inflow (job loss)}} \;-\; \underbrace{p(\theta)\,u}_{\text{outflow (job finding)}}, \qquad e = 1 - u.$$

Setting $\dot{u} = 0$:

Steady-state unemployment rate

$$\boxed{u^* \;=\; \frac{s}{s + p(\theta^*)}}$$

Vacancies: $v^* = \theta^*\, u^*$.

At the benchmark: $u^* \approx 5.5\%$, $v^* \approx 3.5\%$.

The Beveridge curve

The locus of (u, v) pairs consistent with steady state across all θ.

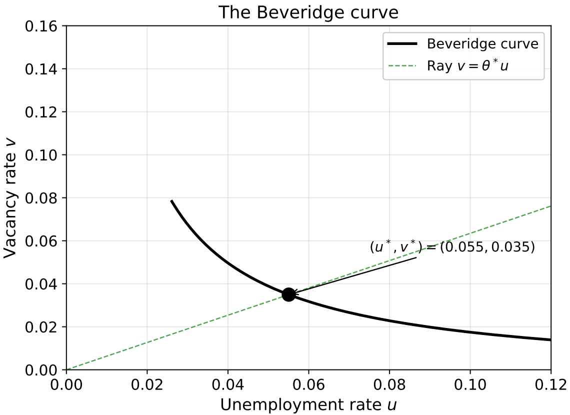
Downward-sloping and convex.

The dashed green ray $v = \theta^* u$ has slope θ^*: all points on it share the same tightness.

Shifts:

- Higher s or lower $\phi \Rightarrow$ outward shift.
- Movements along the curve $\Rightarrow$ JC shifts (e.g. productivity).

Live data: the BLS publishes a monthly-updated U.S. Beveridge curve at bls.gov.



The full picture

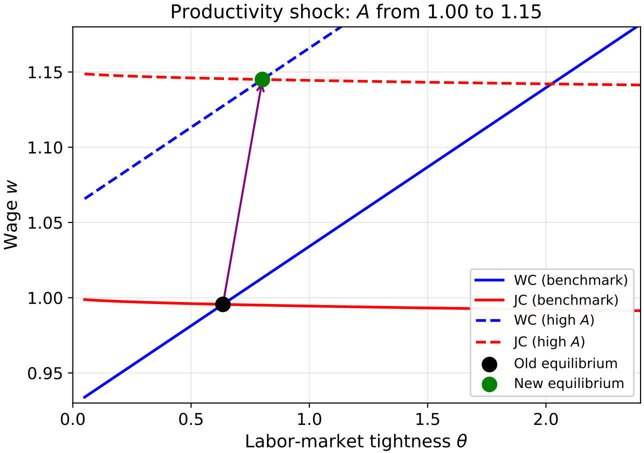
Shock	θ^*	w^*	u^*
Higher productivity A	$\uparrow$	$\uparrow$	$\downarrow$
Higher vacancy cost c	$\downarrow$	ambig.	$\uparrow$
Higher separation rate s	$\downarrow$	$\downarrow$	$\uparrow$
Higher unemployment benefit b	$\downarrow$	$\uparrow$	$\uparrow$
Higher matching efficiency ϕ	$\uparrow$	$\uparrow$	$\downarrow$
Higher bargaining power β	$\downarrow$	$\uparrow$	$\uparrow$

Two families:

- Pro-firm shocks ($A\uparrow$, $c\downarrow$, $s\downarrow$, $\phi\uparrow$): JC shifts up $\Rightarrow$ $\theta^*\uparrow$, $u^*\downarrow$.
- Pro-worker shocks ($b\uparrow$, $\beta\uparrow$): WC shifts up $\Rightarrow$ $w^*\uparrow$, $u^*\uparrow$.

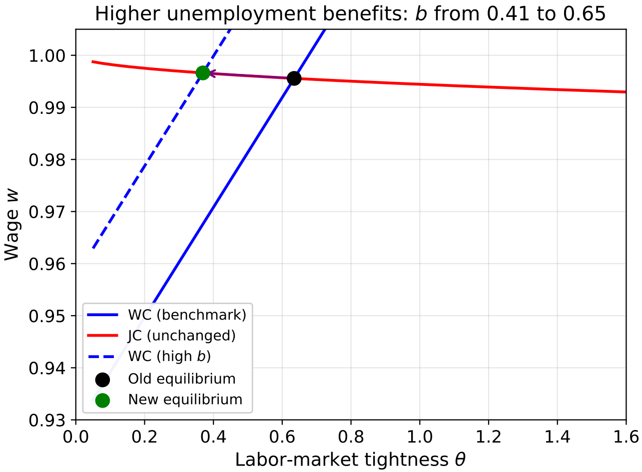
Worked example 1: a productivity shock

$A : 1.00 \to 1.15 \quad \Rightarrow \quad$ both WC and JC shift up $\quad \Rightarrow \quad \theta^* \uparrow,\ w^* \uparrow,\ u^* \downarrow.$



Worked example 2: more generous unemployment benefits

$b : 0.41 \to 0.65 \quad \Rightarrow \quad$ only WC shifts up, JC unchanged $\quad \Rightarrow \quad \theta^* \downarrow,\ w^* \uparrow,\ u^* \uparrow.$



Calibration strategy

Goal: pick parameter values so the model reproduces empirical moments in steady state. Following Shimer (2005): target steady-state unemployment and vacancy rates.

Externally set (data 2002–2025 + literature):

$$s = 0.035, \quad c = 0.12, \quad A = 1, \quad b = 0.41, \quad u_{\text{data}} = 0.05, \quad v_{\text{data}} = 0.035.$$

Internally calibrated (5 unknowns, 5 steady-state equations):

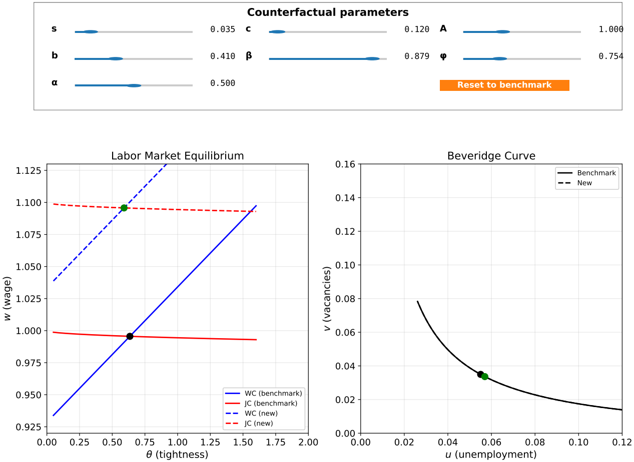
$$\phi, \quad \beta, \quad p(\theta^*), \quad q(\theta^*), \quad w^*.$$

Solved with Excel Solver (or `scipy.brentq`). Result:

$$\phi = 0.754, \quad \beta = 0.879, \quad p^* = 0.601, \quad q^* = 0.946, \quad w^* = 0.996.$$

Caveat on β: $\beta \approx 0.88$ is not a literal estimate of bargaining power. It is what's required for the simple model to match the steady-state targets, partly compensating for omissions (no on-the-job search, no endogenous separations, no wage rigidity). Treat β as a model parameter, not an empirical magnitude.

The Colab simulator



Seven sliders. Solid = benchmark, dashed = counterfactual. Equilibrium dots move in real time.

Suggested labs

Lab 1 — Productivity shocks

Raise A from 1.00 to 1.20. Track θ^*, w^*, u^*. Reproduce the picture from slide 14.

Lab 2 — Unemployment insurance

Raise b from 0.41 to 0.70. Why does only WC shift? Is this consistent with a "moral hazard" story?

Lab 3 — Matching efficiency

Lower ϕ from 0.754 to 0.50. Watch the Beveridge curve shift outward. Compare to the U.S. post-2009.

Lab 4 — Bargaining power

Vary β between 0.30 and 0.95. At which β does $w \to A$? Why?

Where to read more

The framework:

- Mortensen and Pissarides (1994) — foundational paper.
- Pissarides (2000) — canonical textbook treatment.

Calibration:

- Shimer (2005) — target steady-state moments.
- Silva and Toledo (2009) — vacancy posting costs.

The volatility puzzle:

- Shimer (2005) — Nash bargaining $\Rightarrow$ too little volatility.
- Hall (2005) — replace Nash with sticky wages $\Rightarrow$ fixes the puzzle.

What this model leaves out

The framework is deliberately simple. The main omissions, each an active research area:

- **Endogenous separations** (Mortensen & Pissarides, 1994)
- **Worker heterogeneity**
- **On-the-job search**
- **Wage rigidity** (Hall, 2005)
- **Business cycles & aggregate dynamics**
- **Firm dynamics** (entry, exit, heterogeneity)
- **Directed search**
- **Monopsony & labor-market concentration** (Berger, Herkenhoff & Mongey, 2022)
- **Imperfect product-market competition** (Dixit & Stiglitz, 1977; De Loecker, Eeckhout & Unger, 2020)

A natural next step for students who finish this lecture: pick one of these and explore how it modifies the WC–JC equilibrium.

Take-aways

1. Unemployment is structural, not a market failure. It exists because matching takes time.
2. Equilibrium $=$ WC $\cap$ JC. Two curves, one tightness θ^*.
3. θ is the central variable. It hits both sides of the market simultaneously.
4. Comparative statics are graphical. Which curve shifts? In which direction?
5. The interactive simulator turns algebra into experiments. Push a slider, watch the equilibrium move.

Questions?

Companion paper, code, and notebook:

`search_matching_package/`